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LETTER TO THE EDITOR

Is the droplet theory for the Ising spin glass inconsistent with the replica field theory?

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Abstract

Symmetry arguments are used to derive a set of exact identities between irreducible vertex functions for the replica symmetric field theory of the Ising spin glass in zero magnetic field. Their range of applicability spans from mean-field to short-ranged systems in physical dimensions. The replica symmetric theory is unstable for $d > 8$, as in the mean-field theory. For $6 < d < 8$ and $d \lesssim 6$, the resummation of an infinite number of terms is necessary to settle the problem. When $d < 8$, these Ward-like identities must be used to distinguish an Almeida–Thouless line from the replica symmetric droplet phase.

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Field theory has proved to be an extremely useful tool in studying critical transitions in ordinary systems, mostly by providing standard methods such as the loop expansion (above the upper critical dimension) and the renormalization group [1]. Its adaptation to the spin glass problem came just after the introduction of the Edwards–Anderson model and the application of the replica trick [2], resulting in a kind of replica field theory (see [3] and references therein). The first solution of the mean-field theory of the Ising spin glass provided a simple transition in zero external magnetic field from the paramagnet to a replica symmetric (RS) spin glass state [4] which, however, was later proved to be unstable [5]. This instability then persisted perturbatively down to the upper critical dimension 6 [3], and even below it [6]. Repair came soon, at least on the mean-field level, by the famous replica symmetry breaking (RSB) scheme of Parisi (for details see [7]): the spin glass transition is now at a lower symmetry phase which is marginally stable for all $T < T_c$, i.e. it is a massless phase and has a special—ultrametrical—hierarchy. The RSB transition also has the peculiarity of extending to nonzero magnetic fields along the Almeida–Thouless (AT) transition *line* [5].

From this point onwards, the spin glass community has become highly divided about the type of transition and the structure of the spin glass state of short-ranged, finite-dimensional models. Supporters of the RSB scenario followed the classical route trying to build a field theory on the basis of the—highly nontrivial—Parisi solution [6]. As it turned out from the physical interpretation of RSB [7], the Parisi theory corresponds to a complicated ergodicity breaking with a Gibbs state decomposed into many pure states. On the other hand, the so-called droplet theory [8, 9], which was developed mostly on the original lattice system, has a simpler phase structure with only two pure states which are related by the spin inversion symmetry (just like in a ferromagnet), and it predicts that a magnetic field destroys the transition. In the droplet picture, the mode called replicon (R) remains massless in the whole spin glass phase [8] providing the only common feature which both theories share.

The simple phase structure of the droplet theory implies an RS phase with a nonzero order parameter; the corresponding replica field theory has a Lagrangian which is invariant for any permutation of the n replicas (up to cubic order, it was displayed in [10]). Since the coupling constants of such a field theory are chosen by symmetry, or—alternatively—thought to be the outcome of summing out short-ranged fluctuations down to the momentum cut-off Λ, they have little memory of the original control parameters such as temperature and magnetic field. As a nonzero magnetic field does not change the symmetry, it is difficult to distinguish between a zero-field RS spin glass phase and an AT line (massless and replica symmetric too). In this letter, we use exact symmetry arguments to find out the thermal route of the spin glass in zero magnetic field when crossing T_c from the paramagnet having the Lagrangian

$$\mathcal{L} = \frac{1}{2}\sum_{\mathbf{p}}\left(\frac{1}{2}p^2+\bar{m}_1\right)\sum_{\alpha\beta}\phi^{\alpha\beta}_{\mathbf{p}}\phi^{\alpha\beta}_{-\mathbf{p}} - \frac{1}{6\sqrt{N}}{\sum_{\mathbf{p_i}}}'\bar{w}_1\sum_{\alpha\beta\gamma}\phi^{\alpha\beta}_{\mathbf{p}_1}\phi^{\beta\gamma}_{\mathbf{p}_2}\phi^{\gamma\alpha}_{\mathbf{p}_3}$$
$$- \frac{1}{24N}{\sum_{\mathbf{p_i}}}'\Bigg(\bar{u}_1\sum_{\alpha\beta\gamma\delta}\phi^{\alpha\beta}_{\mathbf{p}_1}\phi^{\beta\gamma}_{\mathbf{p}_2}\phi^{\gamma\delta}_{\mathbf{p}_3}\phi^{\delta\alpha}_{\mathbf{p}_4} + \bar{u}_2\sum_{\alpha\beta}\phi^{\alpha\beta}_{\mathbf{p}_1}\phi^{\alpha\beta}_{\mathbf{p}_2}\phi^{\alpha\beta}_{\mathbf{p}_3}\phi^{\alpha\beta}_{\mathbf{p}_4}$$
$$+\bar{u}_3\sum_{\alpha\beta\gamma}\phi^{\alpha\gamma}_{\mathbf{p}_1}\phi^{\alpha\gamma}_{\mathbf{p}_2}\phi^{\beta\gamma}_{\mathbf{p}_3}\phi^{\beta\gamma}_{\mathbf{p}_4} + \bar{u}_4\sum_{\alpha\beta\gamma\delta}\phi^{\alpha\beta}_{\mathbf{p}_1}\phi^{\alpha\beta}_{\mathbf{p}_2}\phi^{\gamma\delta}_{\mathbf{p}_3}\phi^{\gamma\delta}_{\mathbf{p}_4}\Bigg) + \cdots, \tag{1}$$

where the classical fields $\phi^{\alpha\beta}$ are symmetrical in the replica indices α, $\beta = 1, \ldots, n$ with zero diagonal, and momentum conservation is understood in the primed sums. The number of spins N goes to infinity in the thermodynamic limit, while $n \to 0$ ensures the spin glass limit. (A barred notation for the *bare* coupling constants is used to distinguish them from the corresponding *exact* vertex functions.) In addition to the permutational symmetry of the presumed low-temperature phase, the invariants in (1) have the extra attribute that a replica always occurs an even number of times; a Hubbard–Stratonovich-like derivation [3, 10] of (1) makes this point evident. In mathematical form, $\mathcal{L}$ is invariant under the transformation $\phi'^{\alpha\beta} = (-1)^{\alpha+\beta}\phi^{\alpha\beta}$. Following that transformation by the special permutation of grouping odd and even replicas separately—i.e. $(1, 2, 3, \ldots, n) \longmapsto (1, 3, 5, \ldots, 2, 4, 6, \ldots)$—may lead us to figure out that we can define a class of transformations leaving $\mathcal{L}$ of (1) invariant as follows: let us divide the n replicas into two groups consisting of p and $n - p$ elements (p being a free parameter). For the transformed field ϕ', we have

$$\phi'^{\alpha\beta} = \begin{cases} \phi^{\alpha\beta}, & \text{for } \alpha \text{ and } \beta \text{ in the same group,} \\ -\phi^{\alpha\beta}, & \text{for } \alpha \text{ and } \beta \text{ in different groups.} \end{cases} \tag{2}$$

Therefore, the paramagnetic phase has a higher symmetry than even the simplest, generic replica symmetric, spin glass phase, and the presumed paramagnet to droplet spin glass transition breaks that higher symmetry in the replica field theory.

Proceeding further, we can follow the steps of [11] to conclude *exact* identities between the irreducible vertices of the low-temperature RS phase. Including a source term $-\sum_{\alpha\beta} h_{\alpha\beta}\phi^{\alpha\beta}_{\mathbf{p}=0}$ into (1), a Legendre-transformed free energy $F(\mathbf{q})$ can be derived, and it is invariant under the obviously orthogonal transformation $\mathbf{O}$ of equation (2).[1] The derivatives of F provide the zero-momentum one-particle irreducible vertices [1]; their definitions are the following:

$$-H_{\alpha\beta} = \frac{\partial F}{\partial q_{\alpha\beta}}, \qquad M_{\alpha\beta,\gamma\delta} = \frac{\partial^2 F}{\partial q_{\alpha\beta}\partial q_{\gamma\delta}},$$

and similar formulae for $W_{\alpha\beta,\gamma\delta,\mu\nu}$, $U_{\alpha\beta,\gamma\delta,\mu\nu,\rho\omega}$, etc. From $F(\mathbf{q}') = F(\mathbf{q})$ and the orthogonality of $\mathbf{O}$ follows (using tensorial notation):

$$\mathbf{H}' = \mathbf{OH}, \qquad \mathbf{M}' = \mathbf{OMO}^{-1}, \tag{3}$$

and analogous relations for higher order terms. As p is a free parameter, $\mathbf{O}$ is in fact a continuous symmetry transformation, and assuming that $\mathbf{q}$ is replica symmetric, it is easy to derive $\sqrt{(\mathbf{q}'-\mathbf{q})^2} = 2q\sqrt{p(n-p)}$, providing an infinitesimal transformation for $p(n-p)$ small. We can, therefore, expand the left-hand sides of equation (3) around $\mathbf{q}$, and equating the coefficients of identical powers of p and $n-p$. Assuming stationarity, i.e. $\mathbf{H} = 0$, and remembering that we are in an RS state with a nonzero order parameter q, several identities can be derived in this way between the vertices m_1, m_2, m_3; $w_1, \ldots, w_8$; $u_1, \ldots$; etc[2]. These are however—unlike traditional Ward-identities—power series of q with higher and higher order vertices, the most prominent ones are displayed here:

$$m_1 + \frac{n}{2}m_2 = -\left(w_2 + \frac{n}{2}w_3 + \frac{n^2}{2}w_6\right)q + \frac{2}{3}(u_2 + \cdots)q^2 + \cdots, \tag{4}$$

$$m_2 = -\left[\left(w_1 + \frac{1}{3}w_3\right) + n\left(\frac{1}{3}w_5 + w_6\right)\right]q + \cdots, \tag{5}$$

$$m_3 = -\left[\left(w_4 + \frac{1}{2}w_5 - \frac{1}{2}w_6\right) + \frac{n}{2}w_7\right]q + \cdots \tag{6}$$

following from the first and

$$\left(w_2 + \frac{n}{2}w_3 + \frac{n^2}{2}w_6\right) = (u_2 + \cdots)q + \cdots \tag{7}$$

from the second equation in equation (3). (Ellipsis dots in the above expressions are to substitute higher order terms in n or q.)

The value of the identities in equations (4)–(7), and the others not displayed here, rests on their generality—their derivation used only symmetry arguments; as a result, they must be valid for the mean-field as well as for low-dimensional systems. It is tempting to solve these equations iteratively, i.e. assuming all the vertices are analytical in q. The most important result we can get in this way is the famous instability [5] of the replicon mass Γ_R for $n \to 0$ and $u_2 > 0$ at criticality:

$$\Gamma_R = 2m_1 = -\tfrac{2}{3}u_2 q^2 + O(q^3). \tag{8}$$

[1] The $n(n-1)/2$ $q_{\alpha\beta}$'s can be arranged into the vector $\mathbf{q}$, and the transformation in equation (2) may then be written as $\mathbf{q}' = \mathbf{Oq}$ with the diagonal transformation matrix $\mathbf{O}$ having the properties $O_{\alpha\beta,\alpha\beta} = 1$ or -1, depending on α and β belonging to the same group or not.

[2] These identities are much simpler when displayed in terms of the set of vertices with the lower case notation (their bare counterparts are the coupling constants in front of the invariants with the unrestricted sums in the Lagrangian). The linear relationship between these two sets of parameters was derived in [10] for $\bar{m}$ s and $\bar{w}$ s; equations (20)–(24) and table 1. As a property of the generic RS symmetry, it is not restricted to the bare couplings, but is valid for the exact vertices too. The formulae for the 23 quartic couplings are more complicated and will be detailed in a longer publication.

Moreover, all the vertices incompatible with the symmetry of the paramagnetic phase are expressible in terms of those present for $T > T_c$ too. In leading order in q, we have $m_2 = -w_1 q$, $w_2 = u_2 q, w_3 = u_3 q, w_4 = u_4 q$ and $w_5 = u_1 q$, while all the others (m_3, w_6, w_7, w_8) are of order q^2. All these results can be verified for the mean-field theory (for a generic n) using the explicit formulae of [10] and their extensions to the quartic order, and exploiting the fact that bare and exact parameters are identical for a zero-loop calculation.

We cannot follow this procedure in *any* finite dimension d, as the exact vertices are no longer analytical as a function of q. Nevertheless, equations (4)–(7) can now be used perturbatively, for $d > 6$, to compute the *bare* parameters as a function of q, and they will have, besides the analytical part, terms with noninteger, d-dependent powers. Instead of equation (8), we now have for the deviation of the bare replicon mass from its critical value,

$$2(\bar{m}_1 - \bar{m}_{1c}) = -\tfrac{2}{3}\bar{u}_2 q^2 + C_d \bar{w}_1^2 (\bar{w}_1 q)^{1-\frac{\epsilon}{2}}, \tag{9}$$

where $\epsilon = 6 - d$. C_d—unlike the coefficient of the quadratic term—*cannot* be computed by a simple truncation of (4), since a contribution from an arbitrary k-point vertex (multiplied by q^{k-2}) must be included[3].

At that point, two important remarks are appropriate. First, there is some ambiguity in assigning the bare coupling constants to a given physical state below T_c fixed by q: an offset of the zero-momentum fields, $\phi^{\alpha\beta}_{\mathbf{p}=0} \longrightarrow \phi^{\alpha\beta}_{\mathbf{p}=0} - \sqrt{N}\Phi$, leaves all the irreducible vertices unaltered while the bare couplings changing. We can get rid of 'tadpole' insertions by the choice $\Phi = q$ rendering the one-point function zero [3]. We use that case for a unique definition of the bare parameter space throughout this letter. Second, we must emphasize that perturbation theory remains valid for $d > 6$ in the loop-expansion sense, since C_d as well as the coefficient of the q^2 term in (9), and in fact any quantity, can be computed, at least in principle, in terms of $\bar{w}_1^2, \bar{u}_1, \ldots, \bar{u}_4$, etc. Nevertheless, unlike the analytical contributions which can be computed by the truncation method just as in the mean-field case, to get the d-dependent powers at a given order of the loop expansion, we must resum an infinite number of terms (all the 'bubbles' for C_d at one-loop order). (We are not able to do this at the moment, though it may not be a completely hopeless task.)

An evaluation of equation (5) at one-loop level provides us

$$\bar{m}_2 = -\bar{w}_1 q\left[1 + O\left(\bar{w}_1^4\right)\right] + C'_d \bar{w}_1^2 (\bar{w}_1 q)^{1-\frac{\epsilon}{2}}, \tag{10}$$

with C'_d again comprising an infinite number of terms, and there is a good chance that the linear term is exactly $-\bar{w}_1 q$, i.e. it originates from the offset of the fields alone.

After elucidating the q-dependence of the bare parameters, expressions for the exact masses can be straightforwardly computed at the one-loop level:

$$\Gamma_{\mathrm{R}} = -\frac{2}{3}\bar{u}_2 q^2 - 16\bar{w}_1^2 (\bar{w}_1 q)^{1-\frac{\epsilon}{2}} \left[\int \frac{\mathrm{d}^d p}{(2\pi)^d} \frac{1}{p^4 (p^2+2)^2} + \cdots\right] \tag{11}$$

for $n = 0$ and (keeping a generic n now)

$$\Gamma_{\mathrm{A}} - \Gamma_{\mathrm{R}} = (n-2)m_2 = -(n-2)\bar{w}_1 q \left(1 - \frac{n-2}{\epsilon}\bar{w}_1^2\right). \tag{12}$$

The dots in (11) refer to the infinite terms coming from the 5-leg, 6-leg, etc vertices. (Similar terms in (12) are not displayed, as they are definitely subleading when $d > 6$.) While the anomalous mode (A) behaves regularly for any $d > 6$ and is massive, there is a competition between the two terms in Γ_{R}: as long as $d > 8$, the d-dependent power in (11) is subleading, and the truncation method to calculate leading terms works as well as for the mean-field theory.

[3] The one-loop k-leg 'bubble' gives a $(\bar{w}_1 q)^{3-k-\frac{\epsilon}{2}}$ contribution.

We thus conclude that the RS phase is unstable and RSB characterizes the spin glass phase for $d > 8$.

Stability depends on the infinite sum in equation (11), which is not available at the moment, when $6 < d < 8$. We are taking the opportunity to comment on earlier works now. In [3], the second term for Γ_R without the infinite sum represented by the dots was obtained, and—as it is manifestly negative—it was inferred that instability thus persisted below eight dimensions. The authors, however, found a sophisticated way to construct a *different* RS solution which is marginally stable, i.e. $\Gamma_R = 0$. (A modified version of that solution was proposed in a recent paper as a candidate model for the droplet theory [12].) The first result was based on the traditional way to build up a symmetry-broken theory from the symmetrical Lagrangian (1): the bare parameters are continued analytically below T_c, and the offset of the fields, as explained above, then breaks the symmetry. This procedure is obviously correct for the leading terms, e.g. the $O(q)$ term of (10) is reproduced, and it gives definite predictions for the coefficients C_d, C'_d, etc. These, however, should be justified by showing that they satisfy the exact identities above. As a matter of fact, the second procedure of [3] resulting in the massless replicon mode is also correct in leading order; not surprisingly, C_d is tuned to shift Γ_R to zero. The most remarkable observation concerns the inclusion of an external magnetic field H_{ext} which scales as $H_{\text{ext}}^2 \sim q^{2-\frac{\epsilon}{2}}$. It also contributes to C_d and may render the replicon mode massless [13]. The only way to distinguish an AT line from a droplet-like phase is by means of the exact identities which exploit the extra symmetry of the zero-field case.

Finally, we now turn to the $d < 6$ case, where the loop expansion breaks down and the perturbative renormalization group elaborated in [14] takes over its role. What follows from now on is highly based on the details of [14]. To study the crossover region around the zero-field fixed point $\bar{w}_1^{*2} = -\epsilon/(n-2)$, it is useful to introduce Wegner's nonlinear scaling fields [15] defined by the exact renormalization equations $\dot{g}_i = \lambda_i g_i$, where λ_i s are the scaling exponents. The bare parameters can be expressed in terms of g_i s, as displayed here for the three masses (omitting nonlinear terms and irrelevant fields):

$$\begin{aligned}
\bar{m}_1 &= \frac{n-2}{4}\bar{w}_1^{*2} + (g_1 + 2g_2 + g_3) + \cdots,\\
\bar{m}_2 &= -(g_2 + g_3) + \cdots,\\
\bar{m}_3 &= \frac{1}{4}g_3 + \cdots.
\end{aligned}$$

Assuming the form $g_i \cong C_i(\bar{w}_1^* q)^{z_i}$, we can conclude from (6) that $C_3 = O(\epsilon)$, hence hindering us to compute the correction term of $z_3 = 1 + \cdots$ at that order. The other two exponents, however, can be derived from the logarithms of (4) and (5) providing $z_1 = 1 - \epsilon/2 + O(\epsilon^2)$ and $z_2 = 1 + O(\epsilon^2)$. These results are in accord with the relations $\lambda_i = (2 - \epsilon/2 + \eta/2)z_i$—which must follow from the flow of the order parameter $\dot{q} \cong (\beta/\nu)q$, with $\beta/\nu = (2 - \epsilon/2 + \eta/2)$—when comparing with the independent calculations of λ_i s and η in [14]. Although the leading terms for C_i s are trivial [$C_1 = (n-2) + O(\epsilon), C_2 = 1 + O(\epsilon)$ and $C_3 = O(\epsilon)$], the $O(\epsilon)$ corrections can be obtained again, just like for $6 < d < 8$, including all the terms of equations (4)–(7). To see this, let us consider the scaling form of a generic k-point vertex

$$\Gamma^{(k)} = |g_1|^{\frac{1}{\lambda_1}[d-k(2-\frac{\epsilon}{2}+\frac{\eta}{2})]}\tilde{\Gamma}^{(k)}(x, y),$$

where we have used the common notation Γ for the vertices (instead of m's, w's, u's, etc). The two scaling variables are $x = g_2/|g_1|^{\frac{\lambda_2}{\lambda_1}} = g_2/|g_1|^{\frac{z_2}{z_1}}$ and $y = g_3/|g_1|^{\frac{\lambda_3}{\lambda_1}} = g_3/|g_1|^{\frac{z_3}{z_1}}$, whereas $\tilde{\Gamma}^{(k)}$s are scaling functions specific to the given vertex from the k-point family. As we have for the leading terms in ϵ: $\tilde{\Gamma}^{(k)} \sim \bar{w}_1^{*k}$ ($k \geqslant 3$ and forgetting now about the only exception w_1), a typical term of, say, equation (4):

$$q^{k-2}\Gamma^{(k)} \sim q^{k-2}(\bar{w}_1^* q)^{(d\frac{\nu}{\beta}-k)}\bar{w}_1^{*k} \sim \bar{w}_1^{*2}(\bar{w}_1^* q)^{\frac{\gamma}{\beta}},$$

i.e. all terms are of the same ϵ order.

To test stability below six dimensions, $\tilde{\Gamma}_R(x, y)$ must be computed in ϵ-expansion first, and then substituting $x = C_2/|C_1|^{\frac{z_2}{z_1}}$, $y = C_3/|C_1|^{\frac{z_3}{z_1}}$ provides us the replicon mass for $T \lesssim T_c$ in zero magnetic field. From $x = 1/(2-n) + O(\epsilon)$ and $y = O(\epsilon)$, we can conclude

$$\tilde{\Gamma}_R = (-1 + 2x + y) + \cdots = \frac{n}{2-n} + O(\epsilon).$$

In the spin glass limit $n \to 0$, stability depends on the correction term which contains two contributions: the first one comes from the correction of the scaling function $\tilde{\Gamma}_R$—which has been computed and gives a negative (unstable) result, and the second one from those of x and y. These are however—as argued above—not available at the moment, as they result from a resummation of an infinite number of terms in the identity of equation (4).

To conclude, the importance of finding the correct trajectory in the bare parameter space in the absence of an external magnetic field (what is called here the thermal route) is emphasized when the stability of the low-temperature RS phase—the droplet phase—is tested. We argued that for $6 < d < 8$ and $d \lesssim 6$ an infinite number of one-loop graphs should be resummed to settle the problem. Without using the Ward-like identities derived in this letter, the droplet phase cannot be distinguished from an AT line for $d < 8$. Our conclusions are in conflict with those of a recent paper by Moore [16]. Although both of us realized the necessity to resum an infinite number of terms, in [16] self-energy graphs with different *loops* are proposed to be summed to get rid of infrared divergences caused by the replicon mode. In this letter, we argue that allowing for all the one-loop graphs with different *numbers of legs* is a must to ensure the extra symmetry imposed by the lack of an external magnetic field. Old beliefs that the replica field theory is inconsistent with the droplet picture were based on the instability emerging at one-loop order if equation (11) without the infinite terms is used, which we believe is not correct. Therefore, the conclusion that the RS phase is unstable for $6 < d < 8$ is thus premature, just as that it is stable for $d < 6$. Equation (11) loses its relevance for $d < 6$, and the study of the crossover region around the nontrivial fixed point is inevitable. It must be stressed that our identities are valid even in three dimensions, though perturbative methods are not available then to take advantage of them.

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